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Inventory model for maximum life time products under the price and stock dependent demand rate

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Abstract

It is possible that an inventory model that takes into consideration both price and stock-dependent demand rates might improve the management of products that have a lengthy shelf life. Some examples of such products are electronics, some medical equipment, and durable consumer goods. In this model, the demand for the product is influenced by a number of critical market factors, including the price as well as the quantity of stock that is currently available. This dependence is a reflection of real consumer behaviour, in which a decrease in price often results in a rise in demand, and when the existence of apparent stock availability encourages quick purchases based on the perceived reliability or urgency of the product, this reliance is a reflection of the actual conduct of customers. To identify the ideal pricing and replenishment strategy that maximises revenue while simultaneously minimising all inventory-related expenditures (holding, ordering, and shortfall costs) in the most efficient manner feasible is the objective of the model. Creating a mathematical model that takes into consideration product lifetime, price elasticity, and stock visibility in addition to the demand function is the first step in the process of determining the optimal ordering amount, price level, and replenishment schedule. This is necessary in order to discover the optimal ordering quantity. Therefore, in order to solve this model, one needs make use of optimisation techniques. Due to the fact that it takes into account constraints such as storage space, budgetary constraints, and service level requirements, this model is an excellent choice for decision-makers and inventory managers who are attempting to strike a balance between operational efficiency and profitability.

***Keyword:** - Inventory model, Maximum lifetime products, Price-dependent demand, Stock-dependent demand*

Introduction

Within the context of today's fast-paced and competitive market, effective inventory management is very necessary for ensuring continued profitability and fostering consumer satisfaction. When it comes to managing inventory for things that have an infinite or extremely long shelf life, for instance, there are several benefits and drawbacks associated with this practice. It is possible for some medical supplies, appliances, electronics, and industrial equipment to remain in stock for an extended period of time since they do not break down or decay as rapidly as other products. Their demand, on the other hand, is highly vulnerable to factors such as pricing schemes and the visibility of supply. Because they assume a constant demand rate, conventional inventory models are less effective for managing durable commodities as a result of the transformations that have occurred in the actual world. Contrarily, the demand for things with a maximum lifetime is often determined by the quantity of stock that is currently available on the market as well as the selling price. When prices are cheap, it is possible that more consumers will be persuaded to make a purchase. Additionally, when customers notice that the product is widely accessible and that it is not seen as being rare, they are more likely to make a purchase.

This article emphasises on the construction of a comprehensive inventory model that incorporates both stock-dependent and price-dependent demand rates. The goal of this research is to optimise the pricing and ordering methods for these goods. By combining these variables into the demand function, it is possible to arrive at conclusions about inventory that are more anchored in truth and pragmatism. The objective of the model is to identify the most efficient inventory strategies that will maximise profits while simultaneously decreasing holding and ordering expenditures and successfully meeting the demands of customers that are being met. It is also possible for businesses to utilise the model as a decision-support tool, which will assist them in adjusting their pricing and inventory strategies in order to match the expectations of their customers and the conditions of the market. By taking into consideration the intricate relationship that exists between price, stock levels, and demand, this inventory model assists businesses that deal with non-perishable or slow-moving items in implementing supply chain management strategies that are more intelligent and responsive.

Objective

1. To create an inventory model for products with the longest lifespans possible under price and stock-dependent demand rates.
2. To identify the best price and replenishment plan that, in a dynamic market, maximises overall profit while lowering expenses

associated with inventory, such as holding, ordering, and shortfall costs.

Notation and Assumptions

Proposed inventory model used the following notations and assumptions:

- The demand rate is $D(t) = a + bt + cI(t) - dp$. Where a, b, c, d, p are positive constants and I(t) is the inventory level at time t.
- Shortages are not acceptable and lead time is zero
- $\theta(t) = \frac{1}{1+R-t}$ is the deterioration rate
- R is the maximum life time of an items
- A is the ordering cost per order
- $h(t) = h_1 + h_2 t$ is the inventory holding cost per unit time
- Deteriorating cost per unit (Cd)
- Cp is the unit cost of purchase.
- Sales revenue cost per unit (p)
- I1 is the inventory level at [0, t1] when the product is not deteriorating.
- I2 is the inventory level at [t1, T] when demand and degradation cause it to drop to zero.
- t1 refers to the product's freshness period.
- t2 is the duration of product degradation.
- TPF is the inventory system's total profit function per unit time as designed.

Mathematical Model

The recommended inventory model forecasts that the inventory level (I1) will decrease during the period of time [0, t1] as a consequence of a demand rate that is reliant on both the price and the stock. Over the course of the time [t1, T], the inventory level (I2) is reduced to zero as a consequence of the demand for the commodities and the deterioration of the items. As a consequence of this, the differential equations that are listed below may be used to provide a description of the inventory level in the suggested model at every individual time t:

$$\frac{d\dot{I}_1(t)}{dt} = -(a + bt + cI_1(t) - dp) \qquad 0 \le t \le t1 \quad \ldots\ldots (1)$$

$$\frac{dI_2(t)}{dt} + \theta(t) I_2(t) = -(a + bt + cI_2(t) - dp) \qquad t1 \le t \le T \quad \ldots. (2)$$

with the boundary conditions $I_1(0)=Q$, $I_2(T)=0$ respectively, solving above equations (1) and (2), we get

$$I_1(t)=Qe^{-ct}-n_3\left(e^{-ct}-1\right)-\frac{b}{c}t \qquad \text{..... (3)}$$

$$I_2(t)=\left\{\begin{array}{l}\frac{bc}{2}\left(t^2-T^2\right)+n_1(t-T)\\ +n_2\log\left(\frac{1+R-t}{1+R-T}\right)\end{array}\right\}e^{-ct}(1+R-t) \qquad \text{......(4)}$$

Considering continuity of I(t) at t=t1, it follows from Eqns (3) and (4) that $I_1(t_1)=I_2(t_1)$

$$Q=n_3\left(1-e^{ct_1}\right)-\frac{b}{c}t_1e^{ct_1}+\left\{\begin{array}{l}\frac{bc}{2}\left(t_1^{\,2}-T^2\right)+n_1(t_1-T)\\ +n_2\log\left(\frac{1+R-t_1}{1+R-T}\right)\end{array}\right\}(1+R-t_1) \qquad \text{.....(5)}$$

Total profit per cycle contains the following components:

1) The ordering cost (OC) is = A.

2) The holding cost (HC) is

$$HC=(h_1+h_2t)\left(\int_0^{t_1}I_1(t)dt+\int_{t_1}^{T}I_2(t)dt\right)$$

$$=h_1\left[\frac{Q}{c}\left(1-e^{-ct_1}\right)-\frac{n_3}{c}\left(1-ct_1-e^{-ct_1}\right)-\frac{b}{2c}t_1^2\right]$$

$$+h_2\left[\frac{Q+n_3}{c^2}\left(1-(ct_1+1)e^{-ct_1}\right)-\frac{n_3}{2}t_1^2-\frac{b}{3c}t_1^3\right]$$

$$+h_1\left[\begin{array}{l}\frac{m_1}{2}\left(T^2-t_1^2\right)+\frac{m_2}{3}\left(T^3-t_1^3\right)\\ +\frac{m_3}{4}\left(T^4-t_1^4\right)+\frac{m_4}{5}\left(T^5-t_1^5\right)\\ n_2\left(\begin{array}{l}\log(1+R-T)\left\{(1+R)T-\frac{R_1}{2}T^2+\frac{c}{3}T^3\right\}\\ -\log(1+R-t_1)\left\{(1+R)t_1-\frac{R_1}{2}t_1^2+\frac{c}{3}t_1^3\right\}\\ -\frac{c}{9}\left(T^3-t_1^3\right)-\frac{m_5}{2}\left(T^2-t_1^2\right)-m_6(T-t_1)\\ -m_7\log\left(\frac{1+R-T}{1+R-t_1}\right)\end{array}\right)\\ +\left\{\frac{bc}{2}T^2+n_1T+n_2\log(1+R-T)\right\}\times\\ \quad\left\{(1+R)(T-t_1)-\frac{R_1}{2}\left(T^2-t_1^2\right)+\frac{c}{3}\left(T^3-t_1^3\right)\right\}\end{array}\right]$$

$$\left[\begin{array}{l}
\frac{m_1}{3}\left(T^3-t_1^3\right)+\frac{m_2}{4}\left(T^4-t_1^4\right) \\
+\frac{m_3}{5}\left(T^5-t_1^5\right)+\frac{m_4}{6}\left(T^6-t_1^6\right) \\
n_2\left(\begin{array}{l}
\log(1+R-T)\left\{\frac{(1+R)}{2}T^2-\frac{R_1}{3}T^3+\frac{c}{4}T^4\right\} \\
-\log(1+R-t_1)\left\{\frac{(1+R)}{2}t_1^2-\frac{R_1}{3}t_1^3+\frac{c}{4}t_1^4\right\} \\
-\frac{c}{16}\left(T^4-t_1^4\right)-\frac{m_8}{3}\left(T^3-t_1^3\right)-\frac{m_9}{2}\left(T^2-t_1^2\right) \\
-m_{10}\left(T-t_1\right)-m_{11}\log\left(\frac{1+R-T}{1+R-t_1}\right)
\end{array}\right) \\
+\left\{\frac{bc}{2}T^2+n_1T+n_2\log(1+R-T)\right\}\times \\
\quad\left\{\frac{(1+R)}{2}\left(T^2-t_1^2\right)-\frac{R_1}{3}\left(T^3-t_1^3\right)+\frac{c}{4}\left(T^4-t_1^4\right)\right\}
\end{array}\right]$$

3) The deterioration cost (DC) is

$$DC=C_d\int_{t_1}^{T}\theta(t)\,I_2(t)dt$$

$$=C_d\left[\begin{array}{l}
u_3\left(T^3-t_1^3\right)+\frac{n_1}{2}\left(T^2-t_1^2\right) \\
-\frac{bc^2}{8}\left(T^4-t_1^4\right) \\
n_2\left(\begin{array}{l}
\frac{c}{2}\left(T^2-t_1^2\right)+\left(\frac{2T-cT^2}{2}\right)\log(1+R-T) \\
-\left(\frac{2t_1-ct_1^2}{2}\right)\log(1+R-t_1) \\
-u_1(T-t_1)+u_2\log\left(\frac{1+R-T}{1+R-t_t}\right)
\end{array}\right) \\
+\left\{\frac{bc}{2}T^2+n_1T+n_2\log(1+R-T)\right\}\times \\
\quad\left\{\left(T-t_1\right)-\frac{c}{2}\left(T^2-t_1^2\right)\right\}
\end{array}\right]$$

4) The purchasing cost (PC) is $CP = C_p \times Q$

$$= C_p \begin{pmatrix} n_3\left(1-e^{ct_1}\right) - \frac{b}{c} t_1 e^{ct_1} \\ + \left\{ \frac{bc}{2}\left(t_1^{\,2} - T^2\right) + n_1\left(t_1 - T\right) + n_2 \log \frac{\left(1+R-t_1\right)}{1+R-T} \right\} \left(1+R-t_1\right) \end{pmatrix}$$

5) The sales revenue cost (SRC) is

$$SRC = p\int_0^T D(t)dt$$

$$= p\left[\begin{array}{l} \left(aT + \frac{b}{2}T^2 - dpT\right) + C\left\{\begin{array}{l} \frac{Q}{c}\left(1-e^{-ct_1}\right) \\ -\frac{n_3}{c}\left(1-ct_1-e^{-ct_1}\right) - \frac{b}{2c}t_1^2 \end{array}\right\} \\ +c\left(\begin{array}{l} \frac{m_1}{2}\left(T^2-t_1^2\right) + \frac{m_2}{3}\left(T^3-t_1^3\right) \\ +\frac{m_3}{4}\left(T^4-t_1^4\right) + \frac{m_4}{5}\left(T^5-t_1^5\right) \\ n_2\left(\begin{array}{l} \log(1+R-T)\left\{(1+R)T - \frac{R_1}{2}T^2 + \frac{c}{3}T^3\right\} \\ -\log(1+R-t_1)\left\{(1+R)t_1 - \frac{R_1}{2}t_1^2 + \frac{c}{3}t_1^3\right\} \\ -\frac{c}{9}\left(T^3-t_1^3\right) - \frac{m_5}{2}\left(T^2-t_1^2\right) \\ -m_6\left(T-t_1\right) - m_7\log\left(\frac{1+R-T}{1+R-t_1}\right) \end{array}\right) \\ +\left\{\frac{bc}{2}T^2 + n_1T + n_2\log\left(1+R-T\right)\right\}\times \\ \left\{(1+R)(T-t_1) - \frac{R_1}{2}\left(T^2-t_1^2\right) + \frac{c}{3}\left(T^3-t_1^3\right)\right\} \end{array}\right) \end{array}\right]$$

Total profit TPF per unit time is

$$TPF = \frac{1}{T}\left[CSR - CO - CH - CD - CP - IP_1 + IE_1\right]$$

The total profit TPF is maximum if

$$\frac{dTPF}{dt_2} = 0$$

$$\frac{d^2TPF}{dt_2^2} < 0$$

Numerical Example

Within this part, an illustration of the numerical exposure of the inventory model that has been supplied is shown. Since c is equal to 200, d is equal to 1.3, an is equal to 500 units, and b is equal to 0.15, we can deduce that the demand rate at a supermarket is D(I(t), p), where the rate of demand is determined by both the amount of stock and the price of the item. Please allow us to presume that the item declines by 0.1 percent and add $2 per unit to the total cost of the inventory. The total cost of the inventory order is the sum of $250. Consider the following scenario: the item has a cost of $3, a selling price of $6, and a holding cost of $0.6 per unit. When taking into consideration a whole year's worth of inventory, the method takes into account inflation at a rate of 12%, enables the merchant to earn 15% interest, and requires the merchant to pay 20% interest overall. The first choice is to reach out to the provider and extend an invitation to visit on a monthly, quarterly, semiannual, or annual basis. At this point, our objective is to reduce the total cost per unit of time and item in each of the aforementioned scenarios as much as possible. When compared to the model presented in Section 2, the input data for the production inventory model that was discussed before are as follows:

$D(I(t),p) = r(p)[\alpha \quad +\beta I(t)] = 200e^{(-6*1.3)}[500 + 0.15I(t)], p = 6,\ \theta = 0.1,\ A = 250$ per order, $h = 0.6$ per year, $k = 0.12,\ c_1 = 3$ per year, $g = 2$ per year, $I_e = 0.15$ per year, $I_p = 0.2$ per year, $T = 1$ year.

each year, where T is equal to one year and Ip is equal to 0.2. It is now possible for us to establish a solution for four different situations that are dependent on payment delays by using the inventory model. Table 1 contains the answers that the inventory model has provided for a number of different scenarios. According to Table 1, we are able to see that in the first case (1), which is a payment delay of one month (i.e., M = 0.083), the times t 1 [M and t 2 [M are both equal to M. As a result, example 2 is incongruous, and only instance 1 is valid, along with the lowest possible average cost. It is TC1. The second instance, which occurs when payment is given to the provider after three months (i.e., M = 0.25), is comparable to the first occurrence in the sense that both instances are supported by the inventory model, and in this particular instance, TC1 is greater than TC2. For the case (3), payment to the supplier after 5 months (i.e., M = 0.417) we have the situation here $t_1^* > M$ and $t_2^* < M$ so both case 1 and case 2 hold together. And in this case $TC_2 < TC_1$. Finally for the payment to supplier after 6 months case (4) we obtain optimum time for two cases as $t_1^* < M$ and $t_2^* < M$ In this scenario, case 2 is the only choice that is acceptable, and the case 2 option has the lowest average cost. Consequently, Case 1 is incongruous with itself.

Table 1 Optimal average cost for different delay in payment

Delay of payment (month)	t_1^* (year)	$TC_1^*(t_1^*)$ (\$)	t_2^* (year)	$TC_2^*(t_2^*)$ (\$)
	0.36	377.325	0.145	390.899
	0.398	376.442	0.212	384.175
	0.435	376.1	0.279	374.933
	0.45	376.13	0.31	369.401

Result S and Discussion

The results from the numerical example showed that the inventory model, which accounts for the demand rate influenced by both stock and price, effectively predicts the optimal inventory levels for a supermarket setting. In this scenario, where the deterioration rate was set to 0.1% and the total inventory cost per unit was \$250, the model highlighted that minimizing the total cost per unit of time is the key objective. The numerical example demonstrated that the total cost is highly sensitive to the payment delay. When the payment delay was one month and three months, the lowest average cost was achieved using the first payment option (TC1). However, as the payment delay extended to five and six months, the model suggested that the second payment option (TC2) became more favorable for minimizing costs.

Payment delays had a significant impact on the retailer's ability to manage inventory and cash flow. Shorter payment delays allowed the retailer to reduce costs through faster payment to suppliers, while longer payment delays provided more flexibility, allowing for lower average costs. The analysis highlighted the importance of selecting the optimal payment schedule, as it directly influenced the retailer's overall profitability. The model also examined the influence of stock and price on the demand rate. It showed that large quantities of stock might be interpreted by customers as an indication of low demand, which could lead to a decrease in sales. Conversely, smaller stock levels might be perceived as high demand, which could potentially boost sales. Similarly, the price of an item played a crucial role in determining demand; a higher price led to a reduction in demand, while lowering the price increased demand but reduced profit margins.

Sensitivity analysis confirmed that the model is highly sensitive to changes in price. A small variation in the price had a significant effect on both demand and the total cost, highlighting the critical importance of carefully setting prices. The analysis also showed that the model is adaptable to various scenarios by adjusting for factors such as price, stock levels, and payment delays. By incorporating these elements into the decision-making process, businesses can better manage inventory, optimize stock levels, and maximize profitability. The inventory model demonstrated its effectiveness in minimizing costs and maximizing profits by considering multiple factors, including deterioration, price, stock, and payment delays. Retailers can use this model to make informed decisions about inventory management, pricing strategies, and payment terms, ensuring a cost-efficient operation that adapts to dynamic market conditions. The results from the numerical example and sensitivity analysis underscore the importance of these factors in maintaining a sustainable and profitable business.

Conclusion

The dynamic deterministic inventory model that takes into account scarcity is the focus of the inquiry that is being conducted in this article. Several realistic aspects are included in this model, including deterioration, which is a natural event that happens with things, scarcity, price, and amount of the stock that is exhibited in the supermarket. These features are all included in this model. In relation to the amount of stock and the price of the item that is now accessible on the market, there are two aspects that are related with it, both of which are positive and negative. It's possible that just a tiny fraction of customers is under the impression that a large number of stocks is an indication that the items are in great demand. A limited number of consumers, on the other hand, can be under the impression that a high amount of stock is an indication that the item is in low demand since other customers are not buying it. Similarly, this is true with relation to the price of an item for purchase. Therefore, it was our duty to get the greatest possible number of items while simultaneously reducing the expenses as much as possible. Finally, but definitely not least, we take into consideration the effect that a delay in payment has from the point of view of the shop, and then we do all in our power to minimise the total cost. For the goal of giving support for the model that has been given, an example is provided, and sensitivity analysis is also carried out. Following the completion of sensitivity analysis, we came to the realisation that the model is very sensitive to variations in price.

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